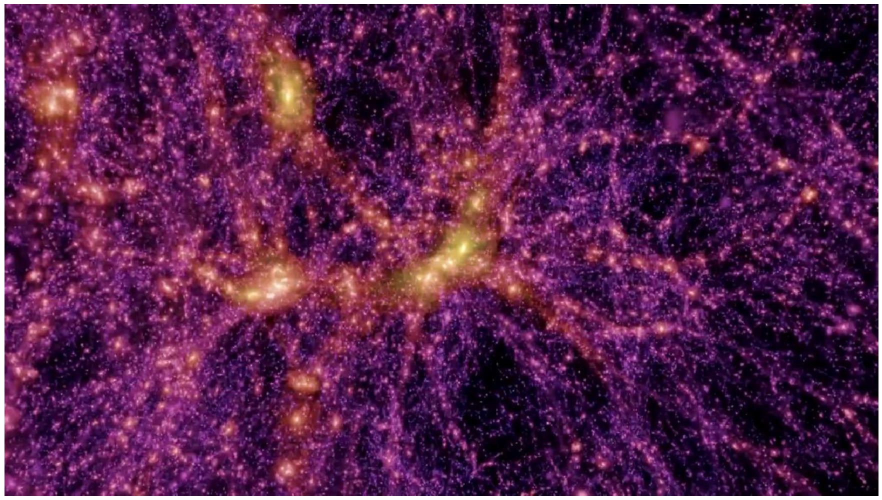
MIT Invitational, Jan 2020

Astronomy C



Competitors: ______________________

School name: ______________________

Team number: ______________________

INSTRUCTIONS

1. Please turn in **all materials** at the end of the event.
2. You may separate the pages, but do not forget to put your **team number** at the top of every page.
3. Copy your multiple choice answers to the answer page.
4. Do not worry about significant figures. Use 3 or more in your answers, regardless of how many are in the question.
5. Please **do not access the internet** during the event. If you do so, your team will be disqualified.
6. Good luck! And may the stars be with you!

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Section B (30 pts)

41. **Gravitational lensing.** Use JS9 to analyze an observation of an Einstein ring around a massive lensing object.

    (a) (2 points) Use JS9 to determine the diameter of the ring in pixels.

    Accept answers between 55 - 65 pixels. Note that the ring has thickness, and the diameter should be measured from the middle of the thickness. Many teams overestimated by (presumably) measuring the outer diameter.

    (b) (2 points) Convert the previous answer to arcseconds. Use the "counts in regions" tool.

    Accept answers between 2.18 - 2.58 arcseconds. JS9 tells you that the conversion (for this image) is 0.03962 arcseconds per pixel, so you just multiply this number by the answer from part a. Full credit given if you do this calculation (even if your answer from part a is wrong).

    (c) (2 points) You determine that the distance to the central object is 9.2 megaparsecs. What is the diameter of the ring, in parsecs? Hint: convert the answer from part b) to radians and use the small angle approximation. (Remember: 3600 arcseconds in a degree.) Ignore cosmological effects.

    Accept answers between 97 - 115.1 parsecs. Partial credit: 1.05×10^{-5} to 1.25×10^{-5} radians. This is NOT the "typical" parallax calculation (which some teams tried to do). We are not determining anything via parallax. Rather, we already have the distance and the angular size; we want to to find the diameter of the ring, no parallax involved. It's just trigonometry with a VERY long isoceles triangle, which we can split into two VERY long right triangles, where the base is the distance to the ring, the height is the ring radius, and the angle is from part b. You have to convert this angle into radians, and then use the small-angle approximation $\tan\theta \approx \theta$ to calculate the height of the triangle.

    (d) (2 points) How are the answers to parts b) and c) related to the mass of the lensing object?

    The mass of the lensing object affects how much light rays "bend" around the object. So there is a relationship between the ring radius (i.e. impact parameter), the mass, and the angular deflection.

42. **Galaxy observations.** Galaxy A has apparent magnitude 7.5 and angular diameter $7'$. Galaxy B has apparent magnitude 5.0 and angular diameter $3°$. Remember that there are 60 arcminutes in a degree.

    (a) (2 points) Find the ratio of the distances to the galaxies d_A/d_B assuming their physical sizes are equal.

    By the small angle formula, we know that $D = \frac{\alpha d}{3438}$ where D represents the physical size of the galaxy, α is the angular diameter in arcminutes, and d is the distance to the galaxy. For ease of computation, we note that $3° = 60' \cdot 3 = 180'$. Hence, $\alpha_A d_A = \alpha_B d_B$, $\frac{d_A}{d_B} = \frac{\alpha_B}{\alpha_A} = \frac{180}{7} = 25.7$. Thus, the ratio between their distances is 25.7:1.

    (b) (2 points) Find the ratio of the distances to the galaxies d_A/d_B assuming their absolute magnitudes are equal.

    Since the galaxies have the same absolute brightness, we know their absolute magnitudes $M_A = M_B$. Hence, we have the system of equations $7.5 - M = 5\log_{10}(d_A) - 5$ and $5.0 - M = 5\log_{10}(d_B) - 5$. Subtracting these two yields $2.5 = 5\log_{10}\left(\frac{d_A}{d_B}\right)$. Solving yields $d_A/d_B = 3.16$.

    (c) (2 points) Suppose these two galaxies are viewed along the same line of sight. What is the apparent magnitude of their combination?

    We will first determine the total flux observed and then convert this back into magnitudes. Note that $F_A + F_B = 10^{-0.4m_A} + 10^{-0.4m_B}$. So $m_{AB} = -2.5\log_{10}\left(10^{-0.4m_A} + 10^{-0.4m_B}\right)$. Plugging in the given values yields $m_{AB} = 4.90$.

(d) (2 points) Suppose you wanted to view galaxy A in the 1.5 GHz wavelength. Assuming a single dish, how large a diameter would you need?

We know that $\theta = 1.22\frac{\lambda}{D}$ where θ is the angular resolution in radians, λ is the wavelength, and D is the diameter of the detector. We note that $7' = 0.00204$ radians and the corresponding wavelength to 1.5 GHz is 0.20 meters. Hence $D = 120$ meters.

(e) (2 points) Your answer for part d) should have been very large. What's one technique astronomers use to circumvent this difficulty?

Very-long baseline interferometry, i.e. using a bunch of radio telescopes around the earth as one large telescope.

43. **Active Galactic Nuclei.** Consider image 12, which depicts the spectral energy distribution model of an active galaxy from Della Costa *et al.* (in prep.)

(a) (2 points) The average dust temperature in the galaxy is about 36 K. Which of the curves likely represents the luminosity due to dust reradiation?

Red curve. Using Wien's Law yields a peak of about 80 microns, so the only options are the red and black curves. In general with an SED, the uppermost curve (the black curve) represents the total flux of the object, with other curves representing the contribution of an individual component. Partial credit was given if the 80 micron peak was correctly calculated.

(b) (2 points) Silicates absorption troughs typically present themselves around the 10 micron region of the distribution. What Seyfert class would you expect to have a deeper trough? Explain.

Type II. Recall that the distinguishing factor between different Seyfert classes is the orientation at which we view them from earth. In particular, we can view the bright nucleus of a Seyfert I directly and conversely, for Seyfert II's, it's obscured by the disk of the galaxy itself. Silicates are a major component of dust, and since there's more intervening dust in a Seyfert II, we'd expect a deeper trough.

(c) (2 points) In order to learn more about this galaxy, you and your colleague observe it in the infrared and X-ray to try to observe its outer edges. Which would provide likely provide greater resolution images? Would it still be the best choice if there was a large amount of extinction? Explain.

The term resolution doesn't refer to how well we can see something, but rather, to the minimum angular size we can view according to the Rayleigh criterion. Shorter wavelengths exhibit less diffraction, and thus offer the greatest resolution. However, short wavelengths are also more susceptible to extinction. So, it's not a great choice.

(d) (2 points) Often when performing estimations in astronomy, it's useful to consider the dynamical timescale of a system to constrain an object's physical size. It describes the minimum amount of time needed for information to travel from one end of an object to another. Consider the flux measurements shown below, taken of a galactic nucleus during an outburst of two points on the edge of the central body. What is the maximum size of the central body, in AU?

To calculate the maximum size, we assume that the time difference is 24 hours, since the delay time is about 24 hours. Multiplying this by c yields the total potential diameter of the central body as 173 AU. If 12 hours or 36 hours were used, full credit was awarded. This was also carried through to the next part.

(e) (2 points) To get a (very) upper bound of the central black hole's mass, we can use the upper bound calculated above as a bound for the black hole's diameter. Calculate the mass of the black hole using this bound. Is this result plausible? Explain.

We find that the black hole mass is $4.3 \times 10^9 \mathrm{M}_\odot$. This is a reasonable estimate since there exist black holes with greater mass (M87's is a great example at $7 \times 10^9 \mathrm{M}_\odot$).

	Time (hours)	12	24	36	48	60	72
Flux (mJy)	Point A	12.1	12.0	12.2	15.6	16.2	16.4
	Point B	11.9	16.0	16.3	17.0	17.1	17.3

Section C (48 pts)

44. **Cosmic Microwave Background.** The following questions relate to understanding the Cosmic Microwave Background (CMB).

   (a) (2 points) Order the following events in cosmic history: Big Bang, photon decoupling, present day, recombination, reionization. From which one of these do the current CMB photons originate?

   Big Bang, recombination, photon decoupling, reionization, present day **(+1)**. Remember that "recombination" is a misnomer, as this is actually the first time that electrons and protons combine to form atoms in cosmic history. As a result of the loss of ions and the creation of neutral atoms, photons decouple from matter (since photons are electromagnetic radiation and interact much more strongly with charged ions than with neutral atoms). Much afterwards, after the creation of the first O-type stars, matter begins to be ionized again due to the stars' radiation. The CMB photons were ricocheting off of the ions from the pre-decoupling epoch; as soon as the matter become "transparent" to photons (due to decoupling), the photons stream (relatively) freely through the universe until hitting our CMB detectors. So CMB originates from photon decoupling **(+1)**.

   (b) (2 points) Antipodal points on the CMB are $1.96d_{\text{hor}}$ away from each other, but they have the same temperature to within 10^{-5}. Why is this a problem, and how is it conventionally resolved?

   Remember that the horizon distance d_{hor} is the furthest distance light can travel over the age of the universe. So these antipodal points actually *have no knowledge of each other* – in jargon, they are causally disconnected. But if that's the case, how do they "know" to be at the same temperature? This issue is called the horizon problem, and is resolved with inflation **(+1)**

   (c) (2 points) In analyzing the CMB, it's often useful to perform a multipole expansion to analyze the angular structure. What useful information is encoded in the $l = 1$ multipole? Why might this explain why only multipoles with $l > 1$ are plotted in image 13? (Note: in multipole expansion $0 \leq l \leq \infty$.)

   The $l = 1$ moment is the dipole moment, which relates to the temperature difference bewteen one hemisphere and the other. Remember that temperature is being probed by the wavelength of CMB radiation, so what this really means is that one side of the sky is redshifted and the other is blueshifted. This is not a "true" feature of the CMB – instead, this doppler shift tells us the earth's overall motion in the universe **(+1)**. This dipole anisotropy greatly skews the CMB relative to the other "real" temperature fluctuations, so the data point would be way off the graph **(+1)**.

   (d) (2 points) What is the relationship between a) the nonzero temperature fluctuations we see in image 13 at angular separations greater than $\approx 1°$, and b) structure formation in the early universe?

   The fluctuations are thought to be caused by density perturbations in matter (primarily dark matter) **(+1)**. This is the Sachs-Wolfe effect. These density perturbations resulted in gravitational instabilities which gave rise to the first galaxies **(+1)**.

45. **Cosmological parameters.** The expansion of our universe (which we'll assume to be Euclidean, i.e. flat) is governed by the matter-energy content of the universe – in particular, we care about radiation, matter, and dark energy. We quantify the present-day relative amounts of each component with density parameters $\Omega_{r,0}$, $\Omega_{m,0}$, and $\Omega_{\Lambda,0}$ respectively.

   (a) (2 points) Image 14 shows how a variety of observational data constrains the values of a $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$. Each colored region represents the parameter values that are consistent with one type of observation. Estimate the true values of $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$.

$0.25 < \Omega_{m,0} < 0.35$ **(+1)** and $0.65 < \Omega_{\Lambda,0} < 0.75$ **(+1)**. Just look at where the regions overlap.

(b) (2 points) $\Omega_{m,0}$ represents the density of all matter in the universe. Using your estimate above, what is $\Omega_{bary,0}$ (the density parameter of "regular" baryonic matter)?

$0.035 < \Omega_{bary,0} < 0.055$ **(+2)** since about 15% of matter is baryonic (85% is dark matter). So $\Omega_{bary,0} = 0.15\Omega_{m,0}$.

(c) (2 points) The super-important cosmological equation that governs the expansion of the universe is called the Friedmann equation; for a flat universe, it can be written as

$$H(a) = H_0\sqrt{\frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0}}$$

where the Hubble parameter H describes the rate of expansion of the universe as a function of the scale factor a. Recall that a is the size of the universe relative to today, and can be used as a proxy for time (i.e. $a = 0$ at the Big Bang, $a = 1$ right now, and a increases as time goes on). H_0 is our familiar Hubble constant. Using your estimate from part a), which component is most important to the dynamics of the universe today? What about in the very distant future? Justify using the flat-universe Friedmann equation.

Currently, dark energy dominates, since its density parameter is the greatest **(+1)**. The scale factor a is effectively the "size of the universe," defined to be 0 at the Big Bang and 1 today. So, presently, $H(a) = H_0\sqrt{\Omega_{r,0} + \Omega_{m,0} + \Omega_{\Lambda,0}}$. We estimated the density parameters for matter and dark energy in part a, and dark matter clearly won. Radiation is negligible today ($\Omega_{r,0} \approx 8.24 \times 10^{-5}$). In the future dark energy will continue to dominate, since its relative importance will continue to be the greatest (it's a cosmological constant, while the coefficients of all the other parameters decrease to zero as $a \to \infty$) **(+1)**. In the history of the universe, first radiation dominated, then matter dominated, and about 4 billion years ago dark energy started to dominate (and will continue to dominate forever, if the universe is flat).

(d) (2 points) The deceleration parameter q_0 is given by $q_0 = \Omega_{m,0}/2 - \Omega_{\Lambda,0}$. Using your estimate from part a), calculate the present deceleration of this universe, and explain its interpretation.

$-0.6 < q_0 < -0.5$ **(+1)**. It tells us that the universe is accelerating in expansion (negative deceleration = acceleration) **(+1)**. A lot of teams forgot order of operations.

(e) We can use q_0 to calculate distances. In particular, we can approximate the proper distance as

$$d_p(z) = \frac{c}{H_0} z \left(1 - z\left(\frac{1+q_0}{2}\right)\right)$$

at sufficiently low z.

i. (2 points) What is the proper distance, in lightyears, to a quasar at redshift 0.25? You can use $q_0 = -0.55$.

3.40 billion lightyears. The fraction c/H_0 is called the Hubble distance, and is about 14 billion lightyears. It's a good number to know (and will make calculations like this easy).

ii. (2 points) What was the comoving distance to the quasar at the time the light we currently see was emitted from the quasar?

The same as previous answer. Comoving distance is invariant with the expansion of the universe, and is defined as equal to the current proper distance.

iii. (2 points) The measured flux from the quasar, converted to convenient units, is 1.45×10^{20} watts per square lightyear. What is the true luminosity? Account for the expansion of the universe.

Convert to luminosity distance $d_L = (1+z)d_p = 4.25$ billion light years **(+1)**. Use the flux relation and solve for luminosity: $L = f(4\pi d_L^2) = 3.3 \times 10^{40}$ watts **(+1)**.

46. **Galaxy rotation.** A spiral galaxy is observed to have a rotation curve that is approximated well by the following expression:

$$v(r) = 250\left(1 - e^{-r/R}\right)$$

Where $v(r)$ is in km/s, r is the radial distance from the center of the galaxy, measured in kpc, and R is a constant equal to 5 kpc. Assume that the galaxy is viewed edge on.

(a) (2 points) Is the rotation curve given by the expression consistent with what we could expect if the galaxy had dark matter? Why or why not?

Yes, it is consistent with what we would expect. At large radii, we would expect the rotation curve to be constant if there is dark matter (+1). As r increases, $e^{-r/R}$ approaches 0, so $v(r)$ approaches the constant value of 250 km/s (+1).

(b) (2 points) Estimate the mass of the galaxy, in solar masses, enclosed within a radius of 20 kpc.

3×10^{11} solar masses (+2)

(c) (2 points) Show that the angular frequency, Ω, is constant with respect to radius near the center of the galaxy (i.e. $r \ll R$). Use the approximation $1 - e^{-r/R} \approx r/R$.

Using the approximation, $v(r) \approx 250r/R$ for small values of r (+1). Knowing that $\Omega r = v(r)$, Ω must be $250/R$, which is a constant (+1).

47. **Boltzmann Distributions and Spectroscopy.** One of the most notable spectral series in Astronomy is the Balmer series, which results from electrons in the second energy level of hydrogen transitioning to higher levels upon absorbing a photon. *Note: it is possible to answer part (c) without answering parts (a) or (b).*

(a) (2 points) For very cool stars, a student postulates that the thermal energy in the surface of the star is not high enough to promote a significant fraction of electrons from the first energy level of hydrogen to the second. Assuming a surface temperature of 3000 Kelvin, what is the ratio of proportion of electrons in the second energy level to the first energy level (i.e. the Boltzmann Factor) throughout the surface of the star? Hint: the ionization energy of hydrogen is 13.6 eV.

7×10^{-18} (half credit for a guess ± 1 order of magnitude)

(b) (2 points) The student attempts to apply this logic to very hot stars, thinking that the increased thermal energy would cause all the hydrogen atoms to become ionized (i.e. be in the "infinitieth" energy level of the hydrogen atom). However, they are surprised to learn that as the temperature increases without bound to infinity, all possible energy levels of the hydrogen atom become equally populated throughout the surface of the star! Why wouldn't the electrons preferentially occupy the highest energy states?

When we are at low temperatures, states with lower energy are preferentially populated since we are constrained by the finite amount of energy available to the system. However, a system at infinite temperature would also have infinite energy, removing that constraint. So, the system has enough energy to access any state it wants. (+1) No one state is energetically preferred over another, so the electrons will distribute themselves evenly among all possible states. This configuration maximizes the entropy of the system (+1), as there are more ways to arrange electrons evenly throughout all states than putting all of them in the highest energy state.

(c) (2 points) Based on the previous two parts, it is clear that the student's idea is on the right track, but not quite correct. Briefly explain the actual reason why very hot stars and very cool stars do not display strong Balmer lines.

If a star's surface is much hotter than 10,000 K the photons coming from the star's interior have a high enough energy to ionize many of the hydrogen atoms in the surface of the star, preventing any transitions to the second energy level (+1). In the same vein, if a star's surface temperature if much cooler than 10,000 K, then the photons coming from the star's interior will not have enough energy to boost many electrons from first energy level to the second energy level. Only electrons in the second energy level can absorb photons characteristic of the Balmer lines (+1). The student was generally right, but it's the photons coming from the star's interior that makes the difference, not the local thermal energy in the surface of the star.

48. **Galaxy collisions.** In a certain galaxy cluster, the number density of galaxies is 10^{-17} pc^{-3}. Take the average radius of each galaxy to be 25 kpc and the average speed of each member galaxy to be 1000 km/s. Unless otherwise specified, assume that the motion of the galaxies within the cluster is random and linear (i.e. gravitational interactions between the galaxies are negligible).

(a) (2 points) Approximating each galaxy as a sphere, find the mean free path, in parsecs, and mean time, in years, between collisions among the galaxies in this cluster.

$l = 5 \times 10^7$ pc (+1), $t = 5 \times 10^{10}$ years (+1)

(b) (2 points) In hope of getting a more accurate answer, an astronomer decides to model the galaxies not as spheres, but instead as thin disks of radius 25 kpc. One of your collaborators says that since the radius of the disk is the same as that of the sphere, the cross-sectional area is the same, resulting in no change in the mean free path or mean time between collisions. Is your collaborator correct? Why or why not?

In the first model, where the galaxies are spheres, the collisions will "look" the same regardless of the orientation of the galaxies. However, then the galaxies are modelled as disks, their cross-sectional area in the direction of a collision will be different for each galaxy depending on their relative orientations (imagine the case where two disks collide edge-on as opposed to face-on as an extreme). The average cross-sectional would be smaller in the second model (+1), increasing the mean free path and mean time between collisions (+1).

(c) (2 points) Another astronomer decides to improve on the original model by including an attractive gravitational force between the galaxies, still modelling them as spheres of radius 25 kpc. How would this affect the mean free path and mean time between galaxy collisions? Is the effect of the gravitational attraction more significant at higher or lower average velocities? Explain your answers qualitatively.

An attractive force between the galaxies would reduce the mean free path and mean time between collisions (+1). This effect will be more pronounced at lower velocities, when the attractive force between the particles results in a larger fractional change in the velocity ($\Delta v / \langle v \rangle$) of each individual galaxy (+1).

49. (8 points) **Research techniques.** Computational methods and simulations are integral to most realms of modern astrophysics research. For the following eight scenarios, pick a simulation or research technique from the list below that would be best suited to support or reject a related hypothesis.

(a) To simulate and analyze the dynamics of a galaxy merger.

N-body. We really only care about the gravitational interactions between the galaxies, which we can model using a bunch of point masses (representing stars, clouds of gas, dark matter, etc).

(b) To survey and classify recent Gaia data to catalog elliptical galaxies.

Neural Network. They're really good for classification tasks, given a data set.

(c) To research cold dark matter's role in galaxy formation.

SPH. Cold dark matter is sometimes modelled as a non-dissipative fluid, so SPH is a good choice for fluid dynamics.

(d) To predict the neutrino extinction of the intergalactic medium.

Monte Carlo. Monte Carlo is really good for probabilistic models of non-interacting particles. Since neutrinos don't self-interact, and extinction is probabilistic, Monte Carlo is a good bet.

(e) To identify filaments of the cosmic web in sky surveys.

Neural Network. Identifying patterns in data is very good for neural nets.

(f) To model the gravitational dynamics of a globular cluster.

N-body

(g) To explore the role of feedback in producing large scale structures.

SPH. "Feedback" here refers to fluid flows (usually intergalactic gas).

(h) To model the creation of jets in long gamma ray bursts.

GRMHD. GRBs are an electromagnetic phenomenon that occur within the strong gravitational fields of black holes, so we need both GR and MHD.

Some research techniques.

- **General Relativistic magnetohydrodynamics (GRMHD):** simulations that solve five equations of general relativity while studying the magnetic properties and behavior of electrically conducting fluids.
- **N-body:** a simulation of a dynamical system of particles, usually influenced by physical forces.
- **Monte-Carlo:** a simulation that uses randomness to model the probability of different outcomes.
- **Neural Network:** a model that learns from training data.
- **Smoothed-Particle Hydrodynamics (SPH):** a computational method used for simulating the mechanics of fluid flows.